Exercise Class- Probability Review

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Ex.1: Introduction to the concept of a RV

Suppose you flip a fair coin three times independently.

1. *What is the sample space?*
2. *What is the set identified by the event that the number of heads is exactly 2? What is its probability?*
3. *Identify with X a RV that represents the number of heads in the sequence. Graph the PDF and CDF.*
4. *Compute $E(X)$, $Var(X)$.*
5. *Do you expect the average number of heads ($\overline{X}$) in a sequence of three flips to be closer to $E(X)$ after you repeat the experiment 20 or 100 times? Why?*

Ex.2: Moments of a discrete RV

After some years of experience, the instructor of the course in Statistics asks to meet the coordinator of the master to present him the following PDF of X, the number of students who miss his evening class on Fridays:

x	0	1	2	3	4	5	6	7	8	≥ 9
$f(x)$	0.35	0.15	0.1	0.00	0.00	0.05	0.05	0.1	0.2	0

1. *Compute the mean, the median, the mode and the standard deviation.*
2. *Is the distribution symmetric?*
3. *What is your intuition on the kurtosis of X?*

The instructor then shows the PDF of Y, the number of students who miss his evening class on :

x	0	1	2	3	4	5	6	≥ 7
$f(x)$	0.05	0.10	0.20	0.25	0.20	0.15	0.05	0

4. *Compare the distribution with the previous one.*
5. *Based on this comparison, what might be the argument of the instructor?*

Ex.3: Linear functions of a discrete RV

Let X be a discrete random variable with values $x = 0, 1, 2$ and probabilities $Pr(X = 0) = 0.25$, $Pr(X = 1) = 0.5$, and $Pr(X = 2) = 0.25$, respectively.

1. Find $E(X)$.
2. Find $Var(X)$.
3. Find the expected value and variance of $Y = 5X + 2$.

Overview of discrete and continuous RV

	RV	
	Discrete	Continuous
Outcome	finite in number (X=people)	infinitely divisible (X=time)
PDF(X)	Probability distribution function	Probability density function
	is a listing of the values x_i taken by X and the associate prob, p_i	the prob associated with any value is zero, we can only assign positive prob to intervals in the range X
	$p(x_i) = Pr(X = x_i)$ $0 \leq p(x_i) \leq 1$ $\sum_i p(x_i) = 1$	$\int_a^b p(x)dx = Pr(a \leq X \leq b)$ $p(x) \geq 0$ $\int_{-\infty}^{+\infty} p(x)dx = 1$
CDF(X)	Cumulative distribution function $P(x_i) = Pr(X \leq x_i) = \sum_{j<i} p(x_i)$ $p(x_i) = P(x_i) - P(x_{i-1})$	Cumulative density function $P(x_i) = \int_{-\infty}^{x_i} p(x)dx$ $p(x_i) = P'(x_i)$
E(X) Var(X)	$\mu_x = \sum_i x_i p(x_i)$ $\sum_i (x_i - \mu_x)^2 p(x_i)$	$\mu_x = \int xp(x)dx$ $\int (x - \mu_x)^2 p(x)dx$

Ex.4: Describe two RVs, given their joint distribution

Two discrete RVs have the following joint probability distribution:

		Y		
		2	4	6
X	1	1/8	1/4	1/8
	3	1/24	1/4	1/24
	9	1/12	0	1/12

1. *Find the marginal probability distribution of X.*
2. *Find the conditional probability distribution of X given Y=2.*
3. *Find the covariance of X and Y.*
4. *Are X and Y independent?*

Ex.5: Identify the distribution of two RV

Suppose you toss two tetrahedra (regular four-sided polyhedron) independently. Let X denote the number on the first tetrahedron and Y the larger between the numbers on the two tetrahedra.

1. *Detect the joint distributions of X and Y.*
2. *Detect the marginal distributions of Y.*
3. *What do you expect about the sign of covariance? Why?*

Ex.6: Sums of random variables

The firm 'Pippo' holds an investment portfolio consisting of two stocks A and B, with 80% *of her capital invested in A and the remaining 20% in B. Stock A has an expected return of* r_A *= 10% and a standard deviation of* σ_A*=15%. Stock B has an expected return of* $r_B = 17\%$ *with a standard deviation of B = 25%.*

1. *Compute the expected return on the portfolio.*
2. *Compute the standard deviation of the returns on the portfolio assuming that the two stocks' returns are perfectly positively correlated, which is* $Corr(A, B) = 1$.
3. *Compute the standard deviation of the returns on the portfolio assuming that the two stocks returns have a correlation of 0.5.*

Ex. 7: Applying the Normal distribution

The length of life (in years) of a 'StatPhone' is approximately a normal distribution $N(3.1, 2.25)$.

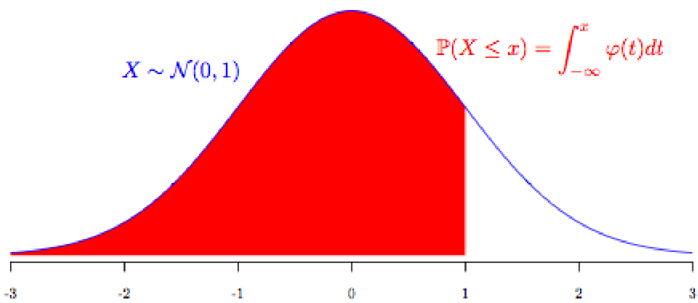
1. *What is the share of phones that will die within the first year?*
2. *What is the share of phones that will survive 4 years or more?*
3. *What fraction of phones will last between 1 and 3.5 years?*
4. *If the manufacturer adopts a warranty policy in which only 10% of the phones have to be replaced, what will be the length of the warranty period?*

Ex. 8: Random sampling

Suppose that X_1, X_2 *and* X_3 *is a sample of observations from a* $N(\mu_X, \sigma_X^2)$ *population, with sample average* $\overline{X}$. *Suppose further that the three observations are not independent, in particular:*

$$Cov(X_1, X_2) = Cov(X_2, X_3) = Cov(X_1, X_3) = 0.5\sigma^2 \tag{1}$$

1. *Find* $E(\overline{X})$.
2. *Find* $Var(\overline{X})$.



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441