Exercise Class - Econometrics Class 3

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Ex.1: Recall what we have already done on heteroskedasticity..
Consider the consumption function:

$$cons_i = \beta_0 + \beta_1 inc_i + u_i \qquad u_i = e_i\sqrt{inc_i} \tag{1}$$

where e_i is a random variable with $E(e_i) = 0$ and $Var(e) = \sigma_e^2$. Assume that e_i is independent of inc_i.

1. *Explain the intuition behind the structure of the error.*
2. *Show that HP3 is satisfied, which is that the zero conditional mean assumption holds.*
3. *Show that HP5 is violated, which is that the error terms and the regressor are not independent.*

Ex.2: Consequences of heteroskedasticity
Discuss which of the following are consequences of heteroskedasticity?

1. The OLS estimators are inconsistent.
2. The usual F statistic no longer has an F distribution.
3. The OLS estimators are no longer BLUE.

Ex.3: Example on inference with different standard errors
The following equation was estimated for the fall and second semester students:

$$\begin{aligned}
\widehat{trmgpa} = \underset{\substack{(.55)\\ [.55]}}{-2.12} &+ \underset{\substack{(.175)\\ [.166]}}{.900}\, crsgpa + \underset{\substack{(.064)\\ [.074]}}{.193}\, cumgpa + \underset{\substack{(.0012)\\ [.0012]}}{.0014}\, tothrs \\
&+ \underset{\substack{(.0002)\\ [.0002]}}{.0018}\, sat - \underset{\substack{(.0018)\\ [.0019]}}{.0039}\, hsperc + \underset{\substack{(.085)\\ [.079]}}{.351}\, female - \underset{\substack{(.098)\\ [.080]}}{.157}\, season \\
&n = 269,\ R^2 = .465.
\end{aligned}$$

Here, trmgpa is term GPA, crsgpa is a weighted average of overall GPA in courses taken, tothrs is total credit hours prior to the semester, sat is SAT score, hsperc is grad- uating percentile in high school class, female is a gender dummy, and season is a dummy variable equal to unity if the student's sport is in season during the fall. The usual and heteroskedasticity-robust standard errors are reported in parentheses and brackets, respectively.

1. Test whether there is an in-season effect on term GPA, using both standard errors. Does the significance level at which the null can be rejected depend on the standard error used?
2. Comment the differences between the standard errors and the robust ones.

Ex.4: Introduction to GLS
Consider a linear model to explain monthly beer consumption

$$beer = \beta_0 + \beta_1 inc + \beta_2 age + \beta_3 educ + \beta_4 female + u$$

with $E(u|inc, age, educ, female) = 0$ and $Var(u|inc, price, educ, female) = \sigma^2 inc^2$. Write the transformed equation that has a homoskedastic error term and comment.

Ex.5: Implication of GLS
Consider the model

$$y_i = \beta_0 + \beta_1 x_i + u_i \tag{2}$$

with heteroskedastic variance $Var(u_i) = \sigma_i^2$ and its transformed homoskedastic version $y_i^* = b_0\sigma_i^{-1} + b_1 x_i^* + u_i^*$ where $y_i^* = \sigma_i^{-1} y_i$, $x_i^* = \sigma_i^{-1} x_i$ and $u_i^* = \sigma_i^{-1} u_i$.

1. Write the formula for the estimator of b_1 and b_0
2. Show that b_0 and b_1 are equal to the least squares estimators β_0 and β_1 when $Var(u_i|x_i) = \sigma^2$. That is, the error variances are constant.
3. Does a comparison of the formulas for b_0 and b_1 with those for β_0 and β_1 suggest an interpretation for the new estimators?

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Ex.6: Example of GLS

Consider the model:

$$y_i = \beta_0 + \beta_1 x_i + u_i \tag{3}$$

where the u_i are independent errors with $E(u_i) = 0$ and $var(u_i) = \sigma^2 x_i^2$. Suppose that you have the following five observations $y = (4, 3, 1, 0, 2)$ and $x = (1, 2, 1, 3, 4)$. Find the generalized least squares estimates of β_0 and β_1.

Ex.7: Clusters, robust and standard errors
You want to estimate the following model:

$$sales_{it} = \beta_0 + \beta_1 solvency\_rate_{it} + u_{it} \tag{4}$$

where i is a firm identifier and t represent the year. Now you estimate the model with artificial data provided by Petersen (2009) through the 'Sandwich' package of R, making different assumption on the standard errors. Below the graph of data and the estimated regression line and the table summarizing the results.

1. Compare the estimated coefficients of running OLS with classical and robust standard errors, have they changed?
2. Compare the robust standard error with the classical ones (do they increase or decrease?). By looking at the graph try to comment whether you would expect that direction of change.
3. Compare the clustered standard errors, which type of effect is predominant in the individual error term?

Figure 1: Scatterplot with regression line

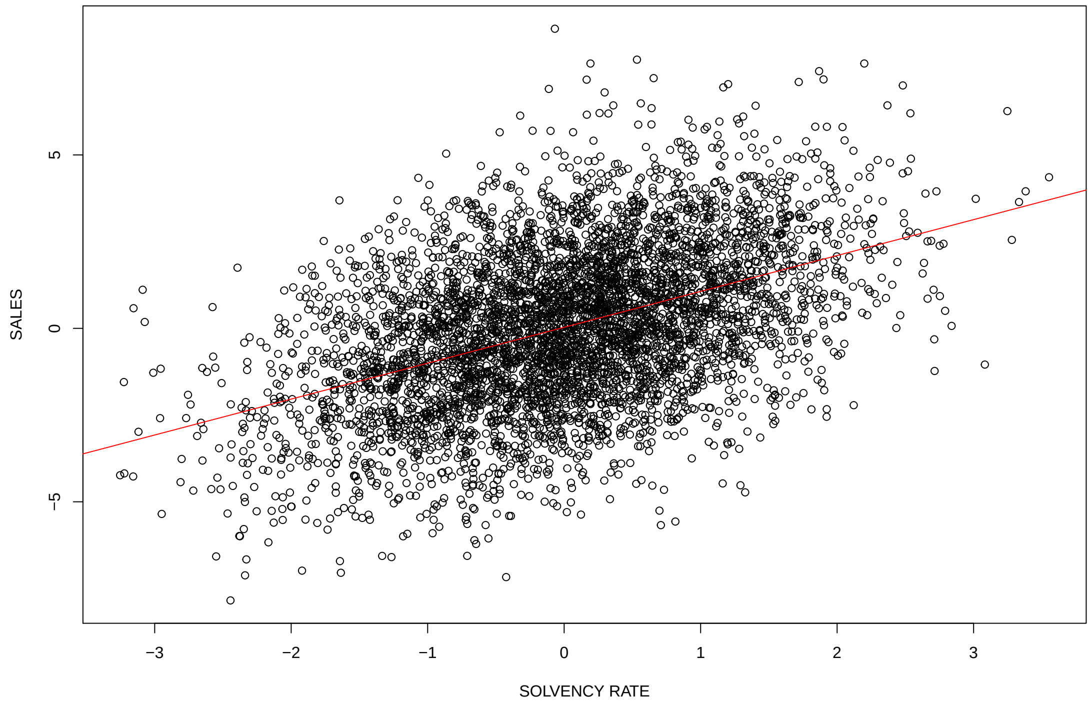

Figure 2: OLS results with classical standard errors

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Call:
lm(formula = sandw$y ~ sandw$x)

Residuals:
    Min      1Q  Median      3Q     Max 
-6.7611 -1.3680 -0.0166  1.3387  8.6779 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.02968    0.02836   1.047    0.295    
sandw$x      1.03483    0.02858  36.204   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.005 on 4998 degrees of freedom
Multiple R-squared:  0.2078,	Adjusted R-squared:  0.2076 
F-statistic:  1311 on 1 and 4998 DF,  p-value: < 2.2e-16
```

Figure 3: OLS results with robust standard errors

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> coeftest(est1, vcov = vcovHC(est1, "HC1"))

t test of coefficients:

            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.029680   0.028361  1.0465   0.2954    
sandw$x     1.034833   0.028395 36.4440   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 4: Standard errors clustered by groups: (i) firm, (ii) years and (ii) firm*year

```
             classical Firm-cluster Year-cluster Firm*Year-cluster
(Intercept) 0.02835932   0.06701270   0.02338672        0.06506392
x           0.02858329   0.05059573   0.03338891        0.05355802
```

Standard Normal Probabilities

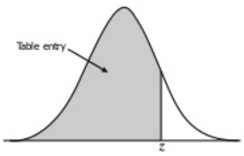

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998